MJ-1(PHYSICS)

Full marks-60

Time-3 hours

Answer the questions as per instruction given.

The figures in the right hand margin indicate marks.

Candidates are required to give answer in their own words as far as possible.

Group-A

(very short answer type questions)

Answer all the following questions.

1. Answer the following questions in a few words. (5×1=5)
   (a) If $\vec{r}$ is the position vector of a point then find the value of $div\, \vec{r}$.
   (b) If $\vec{A}$&$\vec{B}$ are irrotational. Prove that $\vec{A} \times \vec{B}$ is solenoidal.
   (c) Define stress and strain.
   (d) How is the viscosity of a liquid changes with the change in temperature?
   (e) Show that particle having zero rest mass always travels with velocity of light.
2. Short answer type questions. (2×5=10)
   (a) Prove that $curl\ grad\ \varphi = 0$
   (b) Why is a hollow cylinder stronger than a solid cylinder of the same mass, length and material?

Group-B

(Long- answer type questions)

Answer any three questions (15×3=45)

3. (a) State and Prove Gauss's divergence theorem (10+5)
   (b) Prove that $div\ curl\ \vec{A} = 0$
4. (a) Find the expression for curl of a vector field in terms of Cartesian coordinates. (10+5)
   (b) Show that $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\overrightarrow{a}\,\overrightarrow{b}\,\overrightarrow{c}]$.
5. A light beam of rectangular cross-section is resting at its ends on two knife edges (15) And is loaded at its middle. Obtain an expression for the depression produced. How will you determine Young's modulus of elasticity of the beam using it.
6. Derive Poiseuille's formula for capillary flow of liquid. (15)
7. Obtain the formula for the variation of mass with velocity. (15)

Answer:

1. (a) The position vector $\vec{r}$ is written in terms of its Cartesian components as $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

   Now $div\,\vec{r} = \vec{\nabla}.\vec{r} = (\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}).(x\hat{\imath} + y\hat{\jmath} + z\hat{k}) = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}$ =1+1+1=3

   (b) We know $div\,(\vec{A} \times \vec{B}) = \vec{B}.curl\,\vec{A} - \vec{A}.curl\,\vec{B}$

   If $\vec{A}$&$\vec{B}$are irrotational, then $curl\,\vec{A} = 0$ and $curl\,\vec{B} = 0$

   $div\,(\vec{A} \times \vec{B}) = 0$ Hence $\vec{A} \times \vec{B}$ is solenoidal.

   (c) The restoring force per unit area of the body is called stress.

   The ratio of the change in the configuration (i.e. shape, length, or volume) to the original configuration of the body is called strain.

   (d) Increases with the decrease in temperature and vice-versa.

   (e) We know $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

   Or $\sqrt{1 - \frac{v^2}{c^2}} = \frac{m_0}{m}$ in this case m_0=0 (rest mass)

   $\sqrt{1 - \frac{v^2}{c^2}} = 0or1 - \frac{v^2}{c^2}$ =0 or $\frac{v^2}{c^2} = 1$

   Or $v = c$

2. (a) We have $\vec{\nabla}\varphi = (\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z})\varphi = \hat{\imath}\frac{\partial\varphi}{\partial x} + \hat{\jmath}\frac{\partial\varphi}{\partial y} + \hat{k}\frac{\partial\varphi}{\partial z}$

   Now $\vec{\nabla} \times \overrightarrow{\nabla\varphi} = \vec{\nabla} \times (\hat{\imath}\frac{\partial\varphi}{\partial x} + \hat{\jmath}\frac{\partial\varphi}{\partial y} + \hat{k}\frac{\partial\varphi}{\partial z})$

   $$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y} & \frac{\partial\varphi}{\partial z} \end{vmatrix} = \hat{\imath}\left(\frac{\partial^2\varphi}{\partial y\partial z} - \frac{\partial^2\varphi}{\partial z\partial y}\right) + \hat{\jmath}\left(\frac{\partial^2\varphi}{\partial z\partial x} - \frac{\partial^2\varphi}{\partial x\partial z}\right) + \hat{k}\left(\frac{\partial^2\varphi}{\partial x\partial y} - \frac{\partial^2\varphi}{\partial y\partial x}\right)$$

   $= \vec{\nabla} \times \overrightarrow{\nabla\varphi} = 0$

   Provided φ is a perfect differential

   So that $\frac{\partial^2\varphi}{\partial y\partial z} = \frac{\partial^2\varphi}{\partial z\partial y}$ and so on

   Thus $curl\ grad\ \varphi = 0$

   (b) Consider two cylinders one is hollow and another a solid of same length, mass and material.

   Let l = length of each cylinder

   ρ = density of material of both cylinders

   r = radius of solid cylinder

   r_1&r_2 = inner and outer radii of hollow cylinder

   The torsional rigidity of solid cylinder

   $$c = \frac{\pi\eta r^4}{2l}$$

   And torsional rigidity of hollow cylinder

   $$c' = \frac{\pi\eta}{2l}(r_2^4 - r_1^4)$$

$i.e. \frac{c'}{c} = \frac{r_2^4 - r_1^4}{r^4} = \frac{(r_2^2 - r_1^2)(r_2^2 + r_1^2)}{r^4}$

As the cylinders are of the same length & mass.

$$\pi(r_2^2 - r_1^2)l\,\rho = \pi r^2 l \rho$$

$(r_2^2 - r_1^2) = r^2$

$\frac{c'}{c} = \frac{(r_2^2 - r_1^2)(r_2^2 + r_1^2)}{r^4} = \frac{(r_2^2 + r_1^2)}{r^2} > 1$

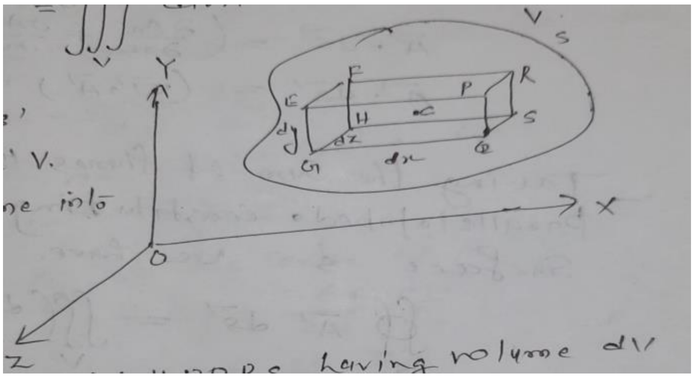
i.e. $c' > c$

Torsional rigidity of hollow cylinder is greater than that of the solid cylinder. Hence the hollow cylinder is stronger than solid cylinder.

3. (a) Statement:-"According to this theorem the surface integral of a vector field $\vec{A}$ over a closed surface $'s'$ is equal to the volume integral of the divergence of a vector field $\vec{A}$ over the volume V enclosed by the surface"

$$\oiint_S \vec{A}\,.d\vec{S} = \iiint_V (\vec{\nabla}\,.\vec{A})\,dV$$

Proof :-



Consider a surface $'s'$ which encloses a volume V . Let us divide this volume into a large no of elementary volumes in the form of parallelepiped. Consider one such parallelepiped EFGHPQRS having volume dV and sides dz . consider a vector $\vec{A}$ at the centre 'c' of a parallelepiped. Let A_x, A_y $\&A_z$ = components of $\vec{A}$ at c along three axes.

The value of $x -$ component of $\vec{A}$ at the centre of face EFGH = $A_x - \frac{\partial A_x}{\partial x}.\frac{dx}{2}$

And that at the centre of face PQRS = $A_x + \frac{\partial A_x}{\partial x}.\frac{dx}{2}$

Since the volume element is infinitesimally small, this component of vector may be considered all over the face. Flux entering the face EFGH = $\left(A_x - \frac{\partial A_x}{\partial x}.\frac{dx}{2}\right) dy\, dz$

Similarly flux leaving the face PQRS = $\left(A_x + \frac{\partial A_x}{\partial x}.\frac{dx}{2}\right) dy\, dz$

Thus net flux leaving the parallelepiped in the $x -$ direction

$= \left[\left(A_x + \frac{\partial A_x}{\partial x}.\frac{dx}{2}\right) - \left(A_x - \frac{\partial A_x}{\partial x}.\frac{dx}{2}\right)\right] dy\, dz = \frac{\partial A_x}{\partial x}\, dx\, dy\, dz$

Similarly, the net flux leaving the parallelepiped in the y & $z -$ directions are $\frac{\partial A_y}{\partial y}\, dx\, dy\, dz$ and $\frac{\partial A_z}{\partial z}\, dx\, dy\, dz$

Total flux of $\vec{A}$ leaving from the parallelepiped

$\vec{A}.\overrightarrow{ds} = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) dx\, dy\, dz = (\vec{\nabla}.\vec{A})dV$ where $div\, \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ and $dV = dx\, dy\, dz$

Taking the sum of fluxes through all the elementary parallelepipeds constituting the volume V of the surface S we have

$$\oiint_S \vec{A}\,.d\vec{S} = \iiint_V (\vec{\nabla}\,.\vec{A})\, dV$$

Proved

(b) $div\, curl\, \vec{A} = \overrightarrow{\nabla}.(\vec{\nabla} \times \vec{A})$

$$= \overrightarrow{\nabla}.\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).\left[\hat{\imath}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\jmath}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\right]$$

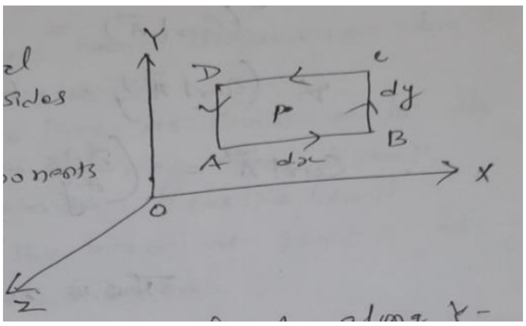
$$\overrightarrow{\nabla}.(\vec{\nabla} \times \vec{A}) = \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} + \frac{\partial^2 A_x}{\partial y \partial z} - \frac{\partial^2 A_z}{\partial y \partial x} + \frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y}$$

=0 assuming that $\vec{A}$ is perfect differential.

Thus $div\, curl\, \vec{A} = 0$.

4. (a)



Consider an infinitesimal rectangular area ABCD of sides dx and dy.

Let A_x , A_y and A_z components of $\vec{A}$ in the direction of X, Y and Z axes at point P.

If the rate of change of A_x along $Y - axis$ is $\frac{\partial A_x}{\partial y}$,

then the value of A_x at the centre of AB

$$= A_x - \frac{\partial A_x}{\partial y}.\frac{dy}{2}$$

Similarly the value of A_x at the centre of CD

$$= A_x + \frac{\partial A_x}{\partial y}.\frac{dy}{2}$$

Again the value of A_y at the centre of BC

$$= A_y + \frac{\partial A_y}{\partial x} \cdot \frac{dx}{2}$$

And the value of A_y at the centre of DA

$$= A_y - \frac{\partial A_y}{\partial x} \cdot \frac{dx}{2}$$

Therefore the line integral along the boundary ABCD

$$=\left(A_x - \frac{\partial A_x}{\partial y} \cdot \frac{dy}{2}\right) dx + \left(A_y + \frac{\partial A_y}{\partial x} \cdot \frac{dx}{2}\right) dy - \left(A_x + \frac{\partial A_x}{\partial y} \cdot \frac{dy}{2}\right) dx - \left(A_y - \frac{\partial A_y}{\partial x} \cdot \frac{dx}{2}\right) dy$$

$$= \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) dx\, dy$$

Now the line integral per unit area is $\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$. This is by definition, is the magnitude of the component of $curl\, \vec{A}$ taken along $Z - axis$.

i.e. $\left(curl\, \vec{A}\right)_z = \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$

similarly the magnitudes of $x\ and\ y$ components of $curl\, \vec{A}$ are

$\left(curl\, \vec{A}\right)_x = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)$ and

$\left(curl\, \vec{A}\right)_y = \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)$

$i.e. curl\, \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) \hat{\imath} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) \hat{\jmath} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \hat{k}$

This is required expression.

(b) $(\vec{a} + \vec{b}).\left[(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})\right] = (\vec{a} + \vec{b}).\left[\vec{b} \times \vec{c} + \vec{c} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a}\right]$

$=(\vec{a} + \vec{b}).\left[\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a}\right]$ as $\vec{c} \times \vec{c} = 0$

$=\vec{a}.(\vec{b} \times \vec{c}) + \vec{a}.(\vec{b} \times \vec{a}) + \vec{a}.(\vec{c} \times \vec{a}) + \vec{b}.(\vec{b} \times \vec{c}) + \vec{b}.(\vec{b} \times \vec{a}) + \vec{b}.(\vec{c} \times \vec{a})$

$=[\vec{a}\, \vec{b}\, \vec{c}] + 0 + 0 + 0 + 0 + [\vec{a}\, \vec{b}\, \vec{c}]$

$=2[\vec{a}\, \vec{b}\, \vec{c}]$

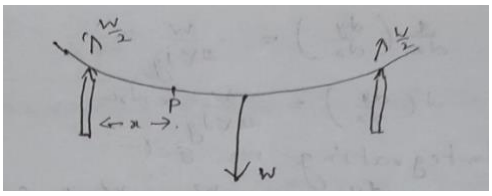
Proved

5. A beam OA is considered. The beam is supported at ends by knife edges $k_1\ and\ k_2$ and is loaded at middle.

Let $l =$ length of beam between two knife edges.

$w =$Load applied at the middle.

$\frac{w}{2} =$Upward reaction at each knife edge.



Basic assumptions

1. The beam is light and hence depression due to its own weight is negligible.
2. The length of beam is large as compare to its cross-section. So that shearing stress is small.
3. The beam bends within the elastic limit.

A cross-section of the beam at point P at distance x from O is considered.

Restoring couple set up at the point P $= \frac{YI_g}{R}$, where $Y =$ Young's modulus of elasticity, I_g =Geometrical moment of inertia, and $R =$ Radius of curvature.

Bending couple at point P= $w\left(\frac{l}{2} - x\right) - \frac{w}{2}(l - x)$

$$= w\left[\frac{l}{2} - x - \frac{l}{2} + \frac{x}{2}\right] = -\frac{wx}{2}$$

At equilibrium $\frac{YI_g}{R} = -\frac{wx}{2}$, or $\frac{1}{R} = -\frac{w}{2YI_g}x$

But we know from calculus $\frac{1}{R} = \frac{d^2y}{dx^2}$, so $\frac{d^2y}{dx^2} = -\frac{w}{2YI_g}x$

Or $\frac{d}{dx}\left(\frac{dy}{dx}\right) = -\frac{w}{2YI_g}x$

Or $d\left(\frac{dy}{dx}\right) = -\frac{w}{2YI_g}x\,dx$

Integrating we get , $\frac{dy}{dx} = -\frac{w}{2YI_g}\frac{x^2}{2} + c_1$,   where c_1 =integration constant ,

At $x = \frac{l}{2}$, $\frac{dy}{dx} = 0$,   so $0 = -\frac{w}{4YI_g}\frac{l^2}{4} + c_1$,   or $c_1 = \frac{wl^2}{16YI_g}$

Putting the value of c_1 in the above equation

$$\frac{dy}{dx} = -\frac{wx^2}{4YI_g} + \frac{wl^2}{16YI_g}$$

$$dy = \left[-\frac{wx^2}{4YI_g} + \frac{wl^2}{16YI_g}\right]dx$$

Again integrating we get

$y = -\frac{w}{4YI_g}\frac{x^3}{3} + \frac{wl^2}{16YI_g}.x + c_2$, where c_2 =integration constant , at $x = 0, y = 0$ & $c_2 = 0$

$$y = -\frac{w}{4YI_g}\frac{x^3}{3} + \frac{wl^2}{16YI_g}.x$$

This gives the depression of the beam at distance x from end O. for depression at middle point i.e at $x = {}^{l}/_{2}$,Let $y = \delta$

$$\delta = -\frac{w}{4YI_g}\frac{l^3}{3 \times 8} + \frac{wl^2}{16YI_g}.\frac{l}{2}$$

$$= \frac{wl^3}{32YI_g}\left[1 - \frac{1}{3}\right] = \frac{wl^3}{48YI_g}$$

$$\delta = \frac{wl^3}{48YI_g}$$

This is the required depression at the middle point of the beam.

Now Young's modulus of elasticity is given by

$Y = \frac{wl^3}{48\delta I_g}$

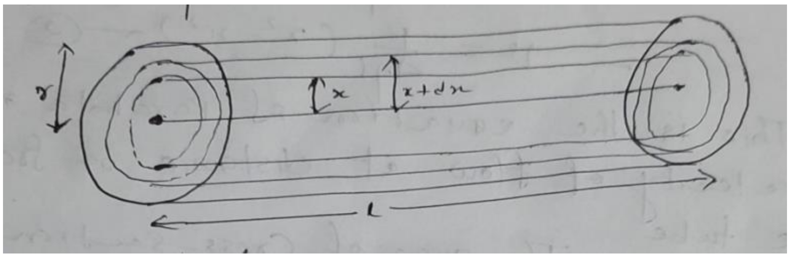
For rectangular cross section of thickness d & $breadth\ b$

Area $a = bd,\ K^2 = \frac{d^2}{12}$,and $I_g = aK^2 = \frac{bd^3}{12}$

$$Y = \frac{wl^3}{48\delta b\frac{d^3}{12}}$$

$$Y = \frac{wl^3}{4\delta bd^3}$$

6.



Consider a capillary tube of length l and radius . Let a liquid be maintained to flow through it.

It is assumed that

(i) The tube is horizontal and so acceleration due to gravity is neglected.

(ii) The motion is stream line and all the stream line flows are parallel to the axis of tube.

(iii) The velocity of liquid along the wall is zero and is maximum along the axis of the tube.

A cylindrical layer of the liquid of radius x and thickness dx is considered.

The viscous force acting on the layer in back ward direction is given by

$F = -\eta A\frac{dv}{dx}$where$\frac{dv}{dx}$ = velocity gradient.

The forward push due to the difference of pressure P on the two sides of the cylinder of radius x is given by $= P\pi x^2$. for steady flow

$-\eta A\frac{dv}{dx} = P\pi x^2$,   but $A = 2\pi xl$, $P\pi x^2 = -\eta 2\pi xl\frac{dv}{dx}$

Or $= -\frac{P}{2\eta l}xdx$, integrating we get $v = -\frac{P}{2\eta l}\frac{x^2}{2} + c$,   where c= integration constant

When $x = r, v = 0$, $c = \frac{P}{4\eta l}r^2$

So $v = \frac{P}{4\eta l}(r^2 - x^2)$

This is the equation of parabola and gives the velocity of flow at distance x from the axis of the tube.

The area of cross-section of the cylindrical layer of radius x and thickness dx is given by

$$= 2\pi xdx$$

Volume of liquid flowing per second through this area $dV = v2\pi xdx$

Hence volume of the liquid flowing out per second through whole tube is given by

$$\int dV = \int_0^r v2\pi xdx$$

$$V = \int_0^r \frac{P}{4\eta l}(r^2 - x^2)\, 2\pi xdx$$

$$= \frac{\pi P}{2\eta l}\int_0^r (r^2x - x^3)dx$$

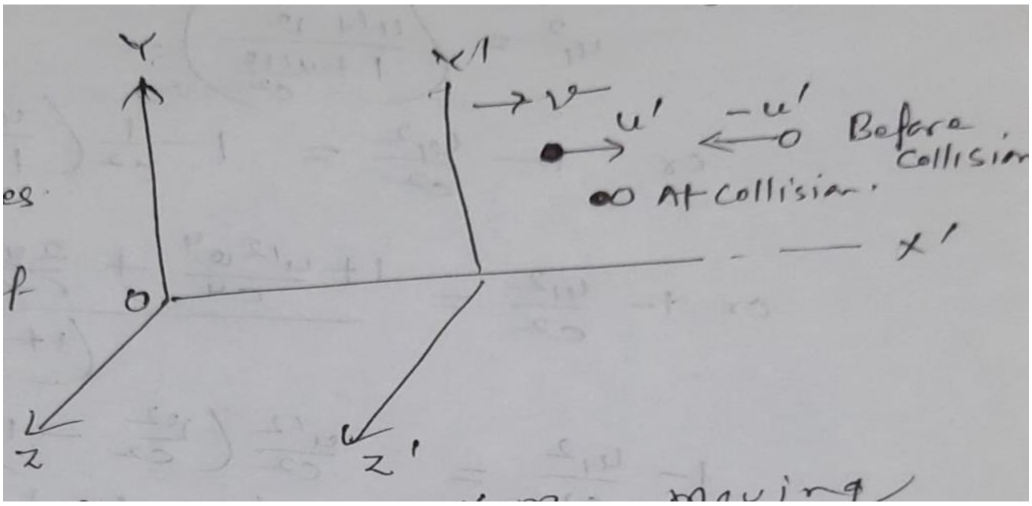
$$= \frac{\pi P}{2\eta l}\left(\frac{r^4}{2} - \frac{r^4}{4}\right)$$

$$V = \frac{\pi P r^4}{8\eta l}$$

This is Poiseuille's formula.

7. According to relativistic idea the mass of a body varies with velocity. It increases with increase of velocity according to relation
   $m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$,   where m_0 =rest mass of the body , c =speed of light,
   v =velocity of the body.

   <u>*Derivation*</u>



Let S & S’ be two inertial frames of references, and v the uniform velocity of S’ with respect to S along $x - axis$.

Let two particles of masses $m_1 and\, m_2$ moving with velocities $u'\ and - u'$ in the frame S’ approaches each other.

The velocities of the particles as seen from the frame S will however be different and are given by the relativistic addition of velocities as, $u_1 = \frac{u'+v}{1+\frac{u'v}{c^2}}$ and $u_2 = \frac{-u'+v}{1-\frac{u'v}{c^2}}$

At the instant of collision, the two particles are momentarily at rest with respect to the frame S’, but as seen from the frame S, they are still moving with velocity v .

Since the total momentum of the two particles is conserved, we have

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)v$$

Or $m_1\left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right) + m_2\left(\frac{-u'+v}{1-\frac{u'v}{c^2}}\right) = (m_1 + m_2)v$

Or $m_1\left(\frac{u'+v}{1+\frac{u'v}{c^2}} - v\right) = m_2\left(v - \frac{-u'+v}{1-\frac{u'v}{c^2}}\right)$

Or $m_1\left(\frac{u'-\frac{u'v}{c^2}}{1+\frac{u'v}{c^2}}\right) = m_2\left(\frac{u'-\frac{u'v}{c^2}}{1-\frac{u'v}{c^2}}\right)$

Or $\frac{m_1}{m_2} = \frac{1+\frac{u'v}{c^2}}{1-\frac{u'v}{c^2}}$

From the above equations

$u_1^2 = \left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right)^2$, or $1-\frac{u_1^2}{c^2} = 1-\frac{1}{c^2}\left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right)^2 = \frac{\left(1+\frac{u'v}{c^2}\right)^2-\left(\frac{u'+v}{c}\right)^2}{\left(1+\frac{u'v}{c^2}\right)^2}$

Or $1-\frac{u_1^2}{c^2} = \frac{1+\frac{u'^2v^2}{c^4}+\frac{2u'v}{c^2}-\frac{u'^2}{c^2}-\frac{v^2}{c^2}-\frac{2u'v}{c^2}}{\left(1+\frac{u'v}{c^2}\right)^2}$

$$1-\frac{u_1^2}{c^2} = \frac{\frac{u'^2}{c^2}\left(\frac{v^2}{c^2}-1\right)-1\left(\frac{v^2}{c^2}-1\right)}{\left(1+\frac{u'v}{c^2}\right)^2} = \frac{\left(1-\frac{u'^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$1+\frac{u'v}{c^2} = \sqrt{\frac{\left(1-\frac{u'^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{u_1^2}{c^2}\right)}}$$

Similarly

$$1-\frac{u'v}{c^2} = \sqrt{\frac{\left(1-\frac{u'^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{u_2^2}{c^2}\right)}}$$

So $\frac{m_1}{m_2} = \frac{\sqrt{1-\frac{u_2^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}}$

$$m_1\sqrt{1-\frac{u_1^2}{c^2}} = m_2\sqrt{1-\frac{u_2^2}{c^2}} = c\ (constant)$$

When $u_1 = 0, m_1 = m_0 = rest\ mass$

$$m_2\sqrt{1-\frac{u_2^2}{c^2}} = m_0 = c$$

Again when $u_2 = 0, m_2 = m_0 = rest\ mass$

$$m_1\sqrt{1-\frac{u_1^2}{c^2}} = m_0 = c$$

in general , if a particle of mass m moving with velocity v , we have

$m\sqrt{1-\frac{v^2}{c^2}} = m_0$ Or $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

This is required relation.